Answer on Question #70844 – Math – Differential Equations

Question

Solve the initial value problem:

$$d^2x/dt^2 - 6dx/dt + 9x = 0,\ x'(0) = 6,\ x(0) = -1\ .$$

Solution

$d^2x/dt^2 - 6dx/dt + 9x = 0$

$\lambda^2 - 6\lambda + 9 = 0$

$\lambda_1 = \lambda_2 = 3$

$x(t) = C_1e^{3t} + C_2e^{3t}t$

$x(0) = C_1 = -1$

$x'(t) = 3C_1e^{3t} + 3C_2e^{3t}t + C_2e^{3t}$

$x'(0) = 3C_1 + C_2 = 6 \quad \Rightarrow \quad C_2 = 6 - 3C_1 = 6+3=9$

$x(t) = -e^{3t} + 9e^{3t}t = e^{3t}(9t-1)$

Answer: $x(t) = e^{3t}(9t-1)$.