

**Answer on Question #70831 – Math – Geometry
Question**

Calculate the arc-length of catenary $\gamma(t) = (t, \cosh t)$ starting at the point $(0, 1)$.

Solution

A parametrized curve in the plane is a differentiable function

$$\gamma(t) = (t, \cosh t),$$

where $t \geq 0$.

Then for any $t > 0$ we define its arclength from 0 to t to be

$$s(t) = \int_0^t \|\gamma'(u)\| du$$

We have $\gamma'(u) = (1, \sinh u)$ and

$$\|\gamma'(u)\| = \sqrt{(1)^2 + (\sinh u)^2} = \sqrt{(\cosh u)^2} = \cosh u$$

$$\begin{aligned} s(t) &= \int_0^t \|\gamma'(u)\| du = \int_0^t \cosh u du = \sinh u \Big|_0^t = \sinh(t) - \sinh(0) = \\ &= \sinh(t), t > 0 \end{aligned}$$

Answer: the arc – length is $s(t) = \sinh(t), t > 0$.