Answer on Question #70827 – Math – Geometry

Question

Calculate the tangent vectors of the following curves:

(i) $\gamma(t) = (\cos^2 t,\ \sin^2 t)$

(ii) $\gamma(t) = (e^t, t^2)$

Solution

A tangent vector is a vector that is tangent to a curve at a given point. For a curve with radius vector $\gamma(t)$, the tangent vector is defined as a derivative with respect to parameter t: $\gamma'(t)$, the unit tangent vector is defined by

$$\mathbf{T}(t) = \frac{\gamma'(t)}{|\gamma'(t)|}$$

(i) For a given curve $\gamma(t) = (\cos^2 t,\ \sin^2 t)$ find $\gamma'(t)$:

$$\gamma'(t) = ((\cos^2 t)',\ (\sin^2 t)') = (2\cos t \cdot (-\sin t),\ 2\sin t \cdot \cos t) = 2(-\sin t\cos t,\ \sin t\cos t)$$

Find $|\gamma'(t)|$:

$$|\gamma'(t)| = 2\sqrt{(-\sin t\cos t)^2 + (\sin t\cos t)^2} = 2\sqrt{2(\sin t\cos t)^2} = 2\sqrt{2}\sin t\cos t$$

Find $\mathbf{T}(t)$:

$$\mathbf{T}(t) = \frac{\gamma'(t)}{|\gamma'(t)|} = \frac{2(-\sin t\cos t,\ \sin t\cos t)}{2\sqrt{2}\sin t\cos t} = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

(ii) For a given curve $\gamma(t) = (e^t, t^2)$ find $\gamma'(t)$:

$$\gamma'(t) = ((e^t)', (t^2)') = (e^t,\ 2t)$$

Find $|\gamma'(t)|$

$$|\gamma'(t)| = \sqrt{(e^t)^2 + (2t)^2} = \sqrt{e^{2t} + 4t^2}$$

Find $\mathbf{T}(t)$

$$\mathbf{T}(t) = \frac{\gamma'(t)}{|\gamma'(t)|} = \frac{(e^t,\ 2t)}{\sqrt{e^{2t} + 4t^2}} = \left(\frac{e^t}{\sqrt{e^{2t} + 4t^2}}, \frac{2t}{\sqrt{e^{2t} + 4t^2}}\right)$$

Answer:

(i) For a curve $\gamma(t) = (\cos^2 t,\ \sin^2 t)$ the tangent vector is

$$\gamma'(t) = 2(-\sin t\cos t, \quad \sin t\cos t)$$

the unit tangent vector is

$$\mathbf{T}(t) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

(ii) For a curve $\gamma(t) = (e^t, t^2)$ the tangent vector is

$$\gamma'(t) = (e^t, 2t)$$

the unit tangent vector is

$$\mathbf{T}(t) = \left(\frac{e^t}{\sqrt{e^{2t} + 4t^2}}, \frac{2t}{\sqrt{e^{2t} + 4t^2}}\right)$$