

ANSWER on Question #70826 – Math – Geometry

QUESTION

Find the cartesian equations of following parametrized curves:

1) $s(t) = (\cos^2 t, \sin^2 t)$

2) $s(t) = (e^t, t^2)$

SOLUTION

1) $s(t) = (\cos^2 t, \sin^2 t)$

$$s(t) = (\cos^2 t, \sin^2 t) \leftrightarrow \begin{cases} x(t) = \cos^2 t \\ y(t) = \sin^2 t \end{cases}$$

As we know

$$\cos^2 x + \sin^2 x = 1, \forall x \in \mathbb{R}$$

(see https://en.wikipedia.org/wiki/Pythagorean_trigonometric_identity)

Then

$$\underbrace{\cos^2 t}_x + \underbrace{\sin^2 t}_y = 1 \leftrightarrow x + y = 1 \leftrightarrow \boxed{y(x) = 1 - x}$$

2) $s(t) = (e^t, t^2) \leftrightarrow \begin{cases} x(t) = e^t \\ y(t) = t^2 \end{cases}$

$$x = e^t \leftrightarrow \ln(x) = \ln(e^t) \leftrightarrow \ln(x) = t \cdot \ln e \leftrightarrow t = \ln(x)$$

Then

$$\begin{cases} t = \ln(x) \\ y(t) = t^2 \end{cases} \leftrightarrow y(x) = (\ln(x))^2 \equiv \ln^2(x) \leftrightarrow \boxed{y(x) = \ln^2 x}$$

ANSWER:

1) $y(x) = 1 - x$; 2) $y(x) = \ln^2 x$

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