

Answer on Question #70823 – Math – Geometry

Question

Find parametrisations of following level curves $y^2 - x^2 = 1$, $x^2/4 + y^2/9 = 1$

Solution

1. Find a parametrisation of the hyperbola $y^2 - x^2 = 1$

$$(y - x)(y + x) = 1$$

$$\begin{cases} y + x = t \\ y - x = \frac{1}{t} \end{cases}$$

$$\begin{cases} y + x + y - x = t + \frac{1}{t} \\ y + x - (y - x) = t - \frac{1}{t} \end{cases}$$

$$\begin{cases} 2y = t + \frac{1}{t} \\ 2x = t - \frac{1}{t} \end{cases}$$

Hence

$$\begin{cases} y = \frac{1}{2} \left(t + \frac{1}{t} \right) \\ x = \frac{1}{2} \left(t - \frac{1}{t} \right) \end{cases}$$

is a parametrisations of the hyperbola $y^2 - x^2 = 1$.

2. Find a parametrisation of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

$$a = 2; b = 3;$$

$$\begin{cases} \frac{x}{2} = \cos t \\ \frac{y}{3} = \sin t \end{cases}$$

Hence

$$\begin{cases} x = 2 \cos t \\ y = 3 \sin t \end{cases}$$

is a parametrisations of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$

Answer:

$$\begin{cases} y = \frac{1}{2} \left(t + \frac{1}{t} \right) \\ x = \frac{1}{2} \left(t - \frac{1}{t} \right) \end{cases} \text{ is a parametrisation of the hyperbola } y^2 - x^2 = 1;$$

$$\begin{cases} x = 2 \cos t \\ y = 3 \sin t \end{cases} \text{ is a parametrisation of the ellipse } \frac{x^2}{4} + \frac{y^2}{9} = 1.$$