Answer on Question #70742 – Math – Differential Equations

Question

1. a) Solve the following ordinary differential equations:

i) dy/dx + 4xy = x
ii) d^2y/dx^2 + 4dy/dx - 12y = cos2x .

Solution

i)

$$y' + 4xy = x$$
$$y' = x(1 - 4y)$$
$$\frac{dy}{1 - 4y} = xdx$$
$$\int \frac{dy}{1 - 4y} = \int xdx$$
$$-\frac{1}{4}\ln(1 - 4y) = \frac{x^2}{2} + C_1$$
$$\ln(1 - 4y) = -2x^2 - 4C_1$$
$$1 - 4y = C_2 e^{-2x^2}$$
$$y(x) = Ce^{-2x^2} + \frac{1}{4},$$

where C_1, C_2, C are real constants.

ii)

$$y'' + 4y' - 12y = cos2x$$

First, solve homogeneous equation:

$$y'' + 4y' - 12y = 0$$

$\lambda^2 + 4\lambda - 12 = 0 \rightarrow \lambda_1 = -6, \lambda_1 = 2$

Thus, the general solution of the homogeneous equation is

$$y_{homogeneous} = C_1 e^{-6x} + C_2 e^{2x},$$

where C_1, C_2 are arbitrary real constants.
Now, let's find any solution for non-homogeneous equation in the form

$$\tilde{y} = Acos2x + Bsin2x$$

If we substitute it in equation, we shall get

$$-4Acos2x - 4Bsin2x - 8Asin2x + 8Bcos2x - 12Acos2x - 12Bsin2x = cos2x$$

As $cos2x$ and $sin2x$ are linearly independent,

$$-4A + 8B - 12A = 1$$
$$-4B - 8A - 12B = 0$$

Solving this system

$$A = -\frac{1}{20}, B = \frac{1}{40}$$

Thus, the general solution of the non-homogeneous equation is

$$y(x) = y_{homogemeous} + \tilde{y} = C_1e^{-6x} + C_2e^{2x} - \frac{1}{20}cos2x + \frac{1}{40}sin2x$$

Answer: i) $y(x) = Ce^{-2x^2} + \frac{1}{4}$; **ii)** $y(x) = C_1e^{-6x} + C_2e^{2x} - \frac{1}{20}cos2x + \frac{1}{40}sin2x.$