## Question

Solve $\sqrt{1-2r} = -2 + \sqrt{4r+9}$.

## Solution

Note that $1-2r \geq 0$ and $4r+9 \geq 0$. So, $r \leq \dfrac{1}{2}$ and $r \geq -\dfrac{9}{4}$.

Then

$$\sqrt{1-2r} = -2 + \sqrt{4r+9},$$

$$\left(\sqrt{1-2r}\right)^2 = \left(-2 + \sqrt{4r+9}\right)^2,$$

$$1-2r = 4 - 4\sqrt{4r+9} + 4r + 9,$$

$$4\sqrt{4r+9} = 6r + 12,$$

$$2\sqrt{4r+9} = 3r + 6,$$

$$4(4r+9) = (3r+6)^2,$$

$$16r + 36 = 9r^2 + 36r + 36,$$

$$16r = 9r^2 + 36r,$$

$$9r^2 + 20r = 0,$$

$$r(9r+20) = 0,$$

$$r = 0,\ 9r = -20,$$

$$r = 0,\ r = -\frac{20}{9}.$$

Since $r = -\dfrac{20}{9} < -\dfrac{9}{4}$, then $r = -\dfrac{20}{9}$ is not a solution.

The solution is $r = 0$.

Checking: $\sqrt{1-2\cdot 0} = -2 + \sqrt{4\cdot 0 + 9}$,

$$\sqrt{1} = -2 + \sqrt{9},$$

$$1 = -2 + 3,$$

$$1 = 1.$$

**Answer:** $r = 0$.