Answer on Question #70541, Math / Geometry

Theorem. Prove that $\alpha'(s)$ and $\alpha''(s)$ are orthogonal.

Poof. Let $\alpha(s)$ be parametrized by arc length. It is well known that $|\alpha'(s)| = 1$. Then $\alpha'(s)\cdot\alpha'(s) = |\alpha'(s)|^2 = 1^2 = 1$. So,

$$\left(\alpha'(s)\cdot\alpha'(s)\right)' = 1',$$

$$\alpha''(s)\cdot\alpha'(s) + \alpha'(s)\cdot\alpha''(s) = 0,$$

$$2\alpha''(s)\cdot\alpha'(s) = 0,$$

$$\alpha''(s)\cdot\alpha'(s) = 0.$$

Since $\alpha''(s)\cdot\alpha'(s) = 0$, $\alpha'(s)$ and $\alpha''(s)$ are orthogonal.