Answer on Question #70540, Math / Geometry

Theorem. Let $\alpha(s)$ be parametrized by arc length. Then $|\alpha'(s)| = 1$.

Poof. Let $\varphi : I \to \mathrm{R}^3$ is a regular curve and the arc length is $s(t) = \int_{t_0}^{t} |\varphi'(t)|\, dt$. We solve for t as $t = t(s)$ to get the function $t : J \to I$. Then

$$\alpha(s) = \varphi(t(s))$$

is parametrized by arc length. So,

$$|\alpha'(s)| = \left|\frac{d\alpha}{ds}\right| = \left|\frac{d\alpha}{dt}\cdot\frac{dt}{ds}\right| = \left|\frac{d\varphi(t(s))}{dt}\cdot\frac{1}{\frac{ds}{dt}}\right| = \left|\frac{d\varphi(t)}{dt}\cdot\frac{1}{\varphi'(t)}\right| = 1.$$