Answer on Question #70527 – Math – Statistics and Probability

QUESTION

All sections of an Introduction to Criminology course at a large university were given the same final exam. Test scores were distributed normally with a mean of 68 and a standard deviation of 6.

a) What percentage of students scored between 60 and 69 (a grade of C) and what percentage scored between 70 and 79 (a grade of B)?

b) In words, briefly summarize your findings.

SOLUTION

a) The probability density of the normal distribution for test scores is

$$f(x) = \frac{1}{\sqrt{2\,\pi\,\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where $\mu = 68$ is the mean of the distribution, $\sigma = 6$ is the standard deviation. A mark is actually a discrete value, but is being approximated by a continuous distribution. That's why we need a continuity correction to calculate the probabilities. So we think of the half-way points between consecutive discrete values.

$$P[60 \le mark \le 69] = \int_{60-0.5}^{69+0.5} f(x)dx \approx 52\%$$

$$P[70 \le mark \le 79] = \int_{70-0.5}^{79+0.5} f(x)dx \approx 37\%$$

Results can be illustrated by figure 1 (a-b), where darker area represents needed probabilities (since areas under a distribution give probabilities).

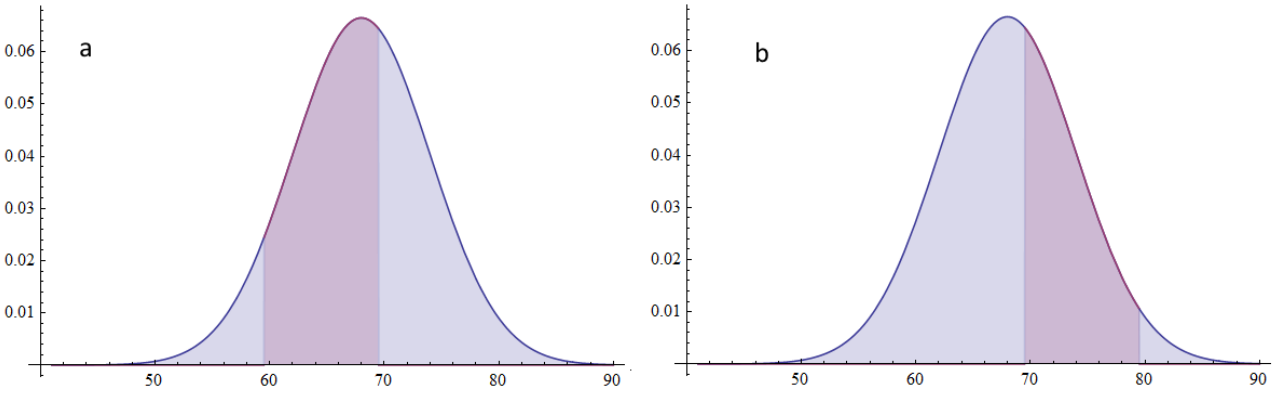

Fig. 1.

ANSWER: a) 52% and 37%.

b) Ranges [60,69] and [70,79] are located near the mean value and each have a size of 10 marks. It is almost 2 times larger than a standard deviation. That is why percentages have such values.

Normal distribution indicates that cheating was either absent or negligible during the final exam. However, a grading system left only 11% for A, D, E, F grades.