Q. Find the length of the astroid $x = a\cos3t, y = a\sin3t, t \in [0, 2\pi]$.

Solution:

$$l = \int_0^{2\pi} \sqrt{[x'(t)]^2 + [y'(t)]^2}\,dt = \int_0^{2\pi} \sqrt{(-3a\sin3t)^2 + (3a\cos3t)^2}\,dt =$$

$$= 3a \int_0^{2\pi} \sqrt{\sin^2 3t + \cos^2 3t}\,dt = 3a \int_0^{2\pi} dt = 6\pi a$$

Answer:

Length of the asteroid equals $6\pi a$.