Answer on Question #70464, Math / Statistics and Probability

Q. Find the length of the astroid $x = acos^3t$, $y = asin^3t$, $0 \le t \le 2\pi$.

Solution.

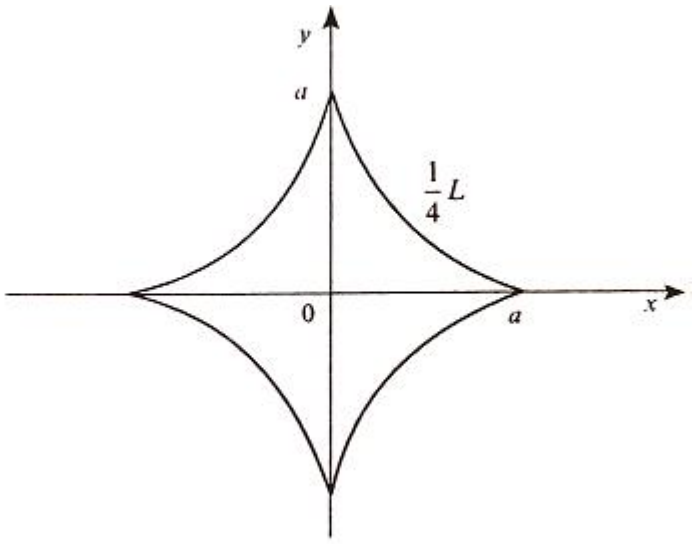

Fig. 1. Astroid.

The length of the curve, given parametrically in interval $t_1 \le t \le t_2$ is calculated by the formula

$$L = \int_{t_1}^{t_2} \sqrt{(x_t')^2 + (y_t')^2}\, dt.$$

So, the length of the astroid is calculated by the formula

$$L = \int_0^{2\pi} 3a \sin t \cos t\, dt.$$

Given the symmetry of astroid (fig. 1), it is enough to find the fourth part of the length of the arc L, which corresponds to a change in the parameter t from 0 to $\frac{\pi}{2}$:

$$\frac{1}{4}L = \int_0^{\frac{\pi}{2}} 3a \sin t \cos t\, dt = \frac{3a}{2}\int_0^{\frac{\pi}{2}} \sin 2t\, dt = \frac{3a}{4}(-\cos 2t)\Big|_0^{\frac{\pi}{2}} = \frac{3a}{4}(1+1) = \frac{3}{2}a.$$

Hence $L = 6a$.

Answer. The length of the astroid equals $6a$.