Answer on Question #70480, Math / Statistics and Probability

A batch of 10 products comes randomly from three factories A, B and C. This batch is checked to detect the defective products. The defective rates of three factories A, B and C are 5%, 10% and 15% respectively

a. Determine the probability that there are exactly two defective products.

b. Given that this batch has two defectives, what is the probability that it comes from factory B.

c. If there are two batches come from the same factory and first one has exactly two defective items, what is the probability that the second one has two defectives?

Solution

Let $X$ be event "there are exactly two defective products". Then

$$P(A) = P(B) = P(C) = \frac{1}{3}$$

$p_A = 0.05, q_A = 1 - p_A = 0.95$

$p_B = 0.1, q_B = 1 - p_B = 0.9$

$p_C = 0.15, q_C = 1 - p_C = 0.85$

$P(X|A) = \binom{10}{2} p_A^2 q_A^8 = 45(0.05)^2(0.95)^8 = 0.074635$

$P(X|B) = \binom{10}{2} p_B^2 q_B^8 = 45(0.1)^2(0.9)^8 = 0.193710$

$P(X|C) = \binom{10}{2} p_C^2 q_C^8 = 45(0.15)^2(0.85)^8 = 0.275897$

(a) Total Probability Rule

$P(X) = P(A)P(X|A) + P(B)P(X|B) + P(C)P(X|C)$

$P(X) = \frac{1}{3}(45)((0.05)^2(0.95)^8 + (0.1)^2(0.9)^8 + (0.15)^2(0.85)^8) = 0.181414$

(b) Bayes' Theorem

$$P(B|X) = \frac{P(B)P(X|B)}{P(X)}$$

$$P(B|X) = \frac{\frac{1}{3}(45)(0.1)^2(0.9)^8}{\frac{1}{3}(45)((0.05)^2(0.95)^8 + (0.1)^2(0.9)^8 + (0.15)^2(0.85)^8)} =$$

$$= 0.355927$$

(c) Let $D$ be event "the second one has two defectives".

i) The probability that two batches come from the factory A, if first one has exactly two defective items

$$P(A|X) = \frac{P(A)P(X|A)}{P(X)}$$

The probability that the second one has two defectives

$$P(A|X)P(X|A) = \frac{P(A)[P(X|A)]^2}{P(X)}$$

ii) The probability that two batches come from the factory B, if first one has exactly two defective items

$$P(B|X) = \frac{P(B)P(X|B)}{P(X)}$$

The probability that the second one has two defectives

$$P(B|X)P(X|B) = \frac{P(B)[P(X|B)]^2}{P(X)}$$

iii) The probability that two batches come from the factory C, if first one has exactly two defective items

$$P(C|X) = \frac{P(C)P(X|C)}{P(X)}$$

The probability that the second one has two defectives

$$P(C|X)P(X|C) = \frac{P(C)[P(X|C)]^2}{P(X)}$$

Total Probability Rule

$$P(D) = P(A|X)P(X|A) + P(B|X)P(X|B) + P(C|X)P(X|C) =$$

$$= \frac{P(A)[P(X|A)]^2 + P(B)[P(X|B)]^2 + P(C)[P(X|C)]^2}{P(X)}$$

$$P(D) = \frac{\frac{1}{3}(45)^2((0.05)^4(0.95)^{16} + (0.1)^4(0.9)^{16} + (0.15)^4(0.85)^{16})}{\frac{1}{3}(45)((0.05)^2(0.95)^8 + (0.1)^2(0.9)^8 + (0.15)^2(0.85)^8)} =$$

$$= 0.219044$$