Answer on Question # 70446 - Math - Geometry

Question

Show that if $\vec{a}$ and $\vec{b}$are in the same direction then $|\vec{a}+\vec{b}| = |\vec{a}| + |\vec{b}|$

Solution. Write $|\vec{a}+\vec{b}|$ as

$$|\vec{a}+\vec{b}| = \sqrt{(\vec{a}+\vec{b})^2} \qquad (1)$$

and find $(\vec{a}+\vec{b})^2$ which is the dot product of a vector $(\vec{a}+\vec{b})$ with itself

$$(\vec{a}+\vec{b})^2 = (\vec{a}+\vec{b}) \cdot (\vec{a}+\vec{b}) = \vec{a}\cdot\vec{a} + \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{a} + \vec{b}\cdot\vec{b}$$

Since the dot product is commutative that is $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ then

$$(\vec{a}+\vec{b})^2 = |\vec{a}|^2 + 2\vec{a}\cdot\vec{b} + |\vec{b}|^2$$

By definition the dot product of $\vec{a}$ and $\vec{b}$ is

$$\vec{a}\cdot\vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\theta$$

where θ is the angle between vectors $\vec{a}$ and $\vec{b}$. Since $\vec{a}$ and $\vec{b}$ are in the same direction then $\theta = 0$ and $\cos 0 = 1$ so $\vec{a}\cdot\vec{b} = |\vec{a}| \cdot |\vec{b}|$. Then we have

$$(\vec{a}+\vec{b})^2 = |\vec{a}|^2 + 2|\vec{a}| \cdot |\vec{b}| + |\vec{b}|^2 = (|\vec{a}| + |\vec{b}|)^2$$

Finally substitute this expression into (1)

$$|\vec{a}+\vec{b}| = \sqrt{(|\vec{a}| + |\vec{b}|)^2} = |\vec{a}| + |\vec{b}|$$

Answer. Thus if $\vec{a}$ and $\vec{b}$ are in the same direction then $|\vec{a}+\vec{b}| = |\vec{a}| + |\vec{b}|$.