Answer on Question #70274 – Math – Trigonometry

Question

sin(2A) + sin(4A) = cos(A) + cos(3A)

Solution

Method 1

$$\sin(2A) - \cos(A) = \cos(3A) - \sin(4A)$$

$$-2\sin(-2A/2-A/2+\pi/4)\sin(-2A/2+A/2+\pi/4) = -(-2\sin(-4A/2-3A/2+\pi/4)\sin(-4A/2+3A/2+\pi/4))$$

$$\sin(-3A/2+\pi/4)\sin(-A/2+\pi/4) = -\sin(-7A/2+\pi/4)\sin(-A/2+\pi/4)$$

$$\sin(-A/2+\pi/4)(\sin(-3A/2+\pi/4)+\sin(-7A/2+\pi/4)) = 0$$

1) $\sin(-A/2+\pi/4)=0$

$$-A/2+\pi/4=\pi n$$

$A=2(\pi/4-\pi n)=\pi/2-2\pi n=(1-4n)\pi/2$

2) $\sin(-3A/2+\pi/4)+\sin(-7A/2+\pi/4)=0$

$2\sin((-3A/2+\pi/4-7A/2+\pi/4)/2)\cos((-3A/2+\pi/4+7A/2-\pi/4)/2)=0$

$2\sin(-5A/2+\pi/4)\cos(A)=0$

$-5A/2+\pi/4=n\pi$ or $A=\pi/2+n\pi$

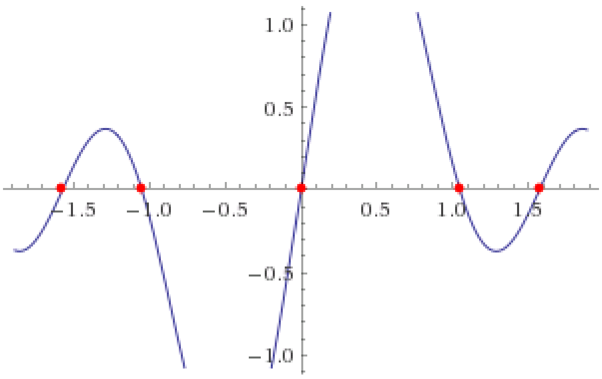
$A=2/5(\pi/4-n\pi)$ or $A=\pi/2+n\pi$

$A=\pi/10-2n\pi/5$ or $A=\pi/2+n\pi$

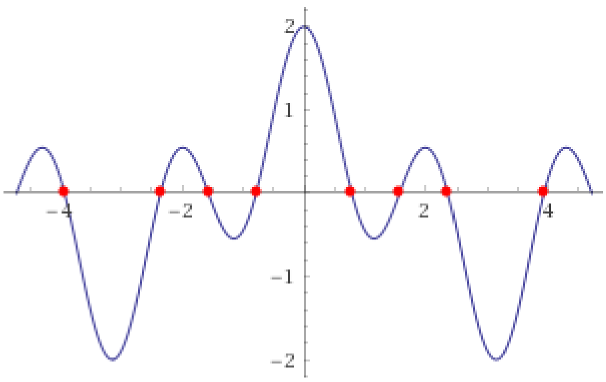
$A=(1-4n)\pi/10$ or $A=(2n+1)\pi/2$

Method 2

$y(A)=\sin(2A)+\sin(4A)$



y(A)=cos(A) + cos(3A)



LHS:

sin(2A)+sin(4A)=sin(2A)+sin2(2A)=
= sin(2A)+2sin(2A)cos(2A)=sin(2A)(1+2cos(2A)) = 2 sin A cos A (1+2cos(2A)) =
= 2 cos A (sin A+2cos(2A)) = 2 cos A (sin A+2(1-2 $\sin^2$ A) sin A) =
= 2 cos A (sin A + 2 sin A – 4 $(\sin A)^3$) = 2 cos A sin(3A)

RHS:

cos(A) + cos(3A)= cos A+4 $\cos^3$ A – 3 cos A =
= 4$\cos^3$ A – 2 cos A = 2 cos A (2 $\cos^2$ A – 1) = 2 cos A cos(2A)

LHS=RHS, so

2 cos A sin(3A) = 2 cos A cos(2A)

If cos A=0 then $\mathrm{A} = -\frac{\pi}{2} + n\pi$

If **cos A ≠ 0 then** we can divide by cos A
sin(3A) = cos(2A) = sin(π/2-2A)

Let's find the corresponding A:

sin(2A+A) = cos(2A)
sin(2A)cos A + cos(2A) sin A = cos(2A)
2 sin A $\cos^2$A + (1- 2 $\sin^2$A)sinA=cos(2A)
2 sin A (1-$\sin^2$ A) + sinA - 2 $\sin^3$A=cos(2A)
2 sin A – 2 $\sin^3$A + sin A – 2 $\sin^3$A=cos(2A)
As you can see
sin(3A)=cos(2A)
3 sin A – 4 $\sin^3$A = 1 – 2 $\sin^2$A

-4$\sin^3$A+2 $\sin^2$A+ 3 sinA – 1 = 0
sin A = t
-(t-1)(4t^2 + 2 t -1)=0

$$D = 2^2 - 4 * 4 * (-1) = 20$$

$$t = \frac{-2 \pm 2\sqrt{5}}{2 * 4} = \frac{-1 \pm \sqrt{5}}{4}$$

$$sin\mathrm{A} = \frac{-1 \pm \sqrt{5}}{4}$$

$sin\mathrm{A} = \frac{-1-\sqrt{5}}{4} \Longrightarrow \mathrm{A} = -\left(-\frac{3\pi}{10}\right) + n\pi$ or $\mathrm{A} = \left(-\frac{3\pi}{10}\right) + n\pi$

$sin\mathrm{A} = \frac{-1+\sqrt{5}}{4} \Longrightarrow \mathrm{A} = -\frac{\pi}{10} + n\pi$ or $\mathrm{A} = \frac{\pi}{10} + n\pi$

Answer: $\mathrm{A} = \frac{3\pi}{10} + n\pi$, $\mathrm{A} = -\frac{3\pi}{10} + n\pi$, $\mathrm{A} = -\frac{\pi}{10} + n\pi$, $\mathrm{A} = \frac{\pi}{10} + n\pi$, $\mathrm{A} = -\frac{\pi}{2} + n\pi$