

Solution to Question #70255, Math / Differential Equations

Question:

Solve the given differential equation by separation of variables.

$$(e^x + e^{-x})dy/dx = y^2$$

Solution:

First let's separate the variables

$$(e^x + e^{-x}) \frac{dy}{dx} = y^2$$

$$\frac{dy}{y^2} = \frac{dx}{(e^x + e^{-x})}$$

Now for the left part

$$\int \frac{dy}{y^2} = -\frac{1}{y} + C_1$$

For the right part

$$\int \frac{dx}{(e^x + e^{-x})} = \int \frac{e^x dx}{((e^x)^2 + 1)}$$

We will set

$$z = e^x$$

Then

$$\int \frac{e^x dx}{((e^x)^2 + 1)} = \int \frac{dz}{(1 + z^2)} = \arctan z + C_2 = \arctan e^x + C_2$$

Finally

$$-\frac{1}{y} + C_1 = \arctan e^x + C_2$$

$$\frac{1}{y} = -\arctan e^x - C^*$$

$$y = -\frac{1}{\arctan e^x + C^*}$$

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