# Answer on Question # 70254 - Math - Differential Equations

**Question**

Determine a region of the xy-plane for which the given DE would have a unique solution whose graph passes through a point (x0,y0) in the region
(y-x)y'=y+x

**Solution**

We have the initial value problem

$$y' = \frac{y+x}{y-x} \qquad y(x_0) = y_0.$$

To determine a region of the xy-plane for which the given DE would have a unique solution whose graph passes through a point $(x_0, y_0)$ in the region, we use the Fundamental Theorem of Existence and Uniqueness for a first order differential equation

$$y' = f(x,y), y(x_0) = y_0.$$

If $f(x,y)$ and $\frac{\partial f}{\partial y}$ are continuous on a rectangular region defined by $a < x < b,\ \ c < y < d$ that contains the point $(x_0, y_0)$, then there exist an interval centered at $x_0$ and a unique function $y(x)$ defined on the interval that satisfied the Initial Value Problem.

For this problem

$$f(x,y) = \frac{y+x}{y-x}$$

then

1. $f(x,y) = \frac{y+x}{y-x}$ is continuous when $y \neq x$ or otherwise $y > x$ or $y < x$.

2. $\frac{\partial f}{\partial y} = \left(\frac{y+x}{y-x}\right)_y = \frac{(y+x)_y \cdot (y-x) - (y+x) \cdot (y-x)_y}{(y-x)^2} = \frac{1 \cdot (y-x) - (y+x) \cdot 1}{(y-x)^2} = \frac{-2x}{(y-x)^2}$

also is continuous when $y > x$ or $y < x$.

Therefore, by the Fundamental Theorem of Existence and Uniqueness, in the region of $y > x$ or $y < x$ the given DE would have a unique solution whose graph passes through a point $(x_0, y_0)$ in the region.

**Answer:** a region of the xy-plane for which the DE $(y-x)y' = y+x$ would have a unique solution whose graph passes through a point $(x_0, y_0)$ in the region, is $y > x$ or $y < x$.