Answer on Question #70253, Math / Differential Equations

Verify that the indicated expression is an implicit solution of the given first order differential equation. Find at least one explicit solution $y = \Phi(x)$ in each case. Use a graphing utility to obtain the graph of an explicit solution.
Give an interval I of definition of each solution Φ.

$$2xydx + (x^2 - y)dy = 0, \quad -2x^2y + y^2 = 1$$

Solution

Implicit differentiation with respect to x

$$\frac{d}{dx}[-2x^2y + y^2 = 1] => -4xy - 2x^2\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$2xy + x^2\frac{dy}{dx} - y\frac{dy}{dx} = 0 => 2xy + (x^2 - y)\frac{dy}{dx} = 0$$

$$2xydx + (x^2 - y)dy = 0$$

Hence, the indicated expression $-2x^2y + y^2 = 1$ is an implicit solution of the given first order differential equation $2xydx + (x^2 - y)dy = 0$.

Find at least one explicit solution $y = \Phi(x)$

$$-2x^2y + y^2 = 1$$

$$y^2 - 2x^2y + x^4 = 1 + x^4$$

$$(y - x^2)^2 = 1 + x^4$$

$$y - x^2 = \pm\sqrt{1 + x^4}$$

$y_1 = x^2 - \sqrt{1 + x^4}$, $y_2 = x^2 + \sqrt{1 + x^4}$