

Answer on Question #70246 – Math – Statistics and Probability

Question

Supposing that the length of certain machine parts may be looked upon as a random variable having a normal distribution with a mean of 2.000 cm, and a standard deviation of 0.050 cm. Specifically we want to test the null hypothesis $\mu = 2.000$ against alternative hypothesis μ not equal to 2.000, on the basis of mean of a random sample of size 30. If probability of type 1 error is 0.05 what is the probability of type 2 error for $\mu=2.010$?

Solution

The test is two-tailed, $\alpha = 0.05$, $Z_{0.025} = 1.96$, and

$$-Z_{\frac{\alpha}{2}} + \sqrt{n} \frac{\mu_0 - \mu_1}{\sigma} = -1.96 + \sqrt{30} \frac{2.000 - 2.010}{0.050} = -3.055,$$

$$Z_{\frac{\alpha}{2}} + \sqrt{n} \frac{\mu_0 - \mu_1}{\sigma} = 1.96 + \sqrt{30} \frac{2.000 - 2.010}{0.050} = 0.865.$$

The power is

$$\gamma(2.010) = P(Z < -3.055) + P(Z > 0.865) = 0.001 + 0.194 = 0.195.$$

So, the Type II error probability is $\beta = 1 - 0.195 = 0.805$.

Answer: 0.805.