Answer on Question # 70218 - Math - Differential Equations

Question

Determine a region of the xy-plane for which the given DE would have a unique solution whose graph passes through a point (x0,y0) in the region

(y-x)y'=y x

Solution

Rewrite the equation in the form

$$y' = \frac{yx}{y-x}, \qquad y(x_0) = y_0.$$

Use the Fundamental Theorem of Existence and Uniqueness for a first order differential equation

$$y' = f(x,y), \qquad y(x_0) = y_0.$$

If $f(x,y)$ and $\frac{\partial f}{\partial y}$ are continuous on a rectangular region defined by $a < x < b,\ \ c < y < d$ that contains the point (x_0, y_0), then there exist an interval centered at x_0 and a unique function $y(x)$ defined on the interval that satisfied the Initial Value Problem $y(x_0) = y_0$.

For this problem $f(x,y) = \frac{yx}{y-x}$ then

1. $f(x,y) = \frac{yx}{y-x}$ is continuous when $y \neq x$, in other words, when $y > x$ or $y < x$.

2. $\frac{\partial f}{\partial y} = \left(\frac{yx}{y-x}\right)_y = \frac{x(y-x)-yx\cdot 1}{(y-x)^2} = \frac{-x^2}{(y-x)^2}$ also is continuous when $y > x$ or $y < x$.

Therefore, by the Fundamental Theorem of Existence and Uniqueness, in the region of $y > x$ or $y < x$ would have a unique solution whose graph passes through a point (x_0, y_0) in the region.

Answer: a region of the xy-plane for which the given DE would have a unique solution whose graph passes through a point (x_0, y_0) in the region is $y > x$ or $y < x$.