# Answer on Question #70217 – Math – Differential Equations

## Question

Solve the initial value problem

$$(e^x + y)dx + (2 + x + ye^y)dy = 0, y(0) = 1.$$

## Solution

Let $P(x,y) = e^x + y$ and $Q(x,y) = 2 + x + ye^y$.

This is a total differential equation $P(x,y)dx + Q(x,y)dy = 0$, because $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} = 1$.

Define $f(x,y)$ such that $\frac{\partial f}{\partial x} = P(x,y)$ and $\frac{\partial f}{\partial y} = Q(x,y)$. Then the solution will be given by $f(x,y) = c_1$, where $c_1$ is an arbitrary real constant.

$\frac{\partial f}{\partial x} = e^x + y \Rightarrow f(x,y) = \int (e^x + y)dx + g(y) = e^x + xy + g(y)$ where $g(y)$ is an arbitrary function of $y$.

Differentiate $f(x,y)$ with respect to $y$ in order to find $g(y)$:

$\frac{\partial f}{\partial y} = x + g'(y) = 2 + x + ye^y \Rightarrow g'(y) = 2 + ye^y \Rightarrow$

$\Rightarrow g(y) = \int (2 + ye^y)dy = 2y + e^y(y-1) + c_2.$

$f(x,y) = e^x + xy + 2y + e^y(y-1) + c_2.$

$e^x + xy + 2y + e^y(y-1) + c_2 = c_1$ or $e^x + xy + 2y + e^y(y-1) = c$

If $y(0) = 1$ then $e^0 + 0 * 1 + 2 * 1 + e^0(1-1) = c \Rightarrow c = 3 \Rightarrow$

$$\Rightarrow e^x + xy + 2y + e^y(y-1) = 3$$

**Answer:** $e^x + xy + 2y + e^y(y-1) = 3.$