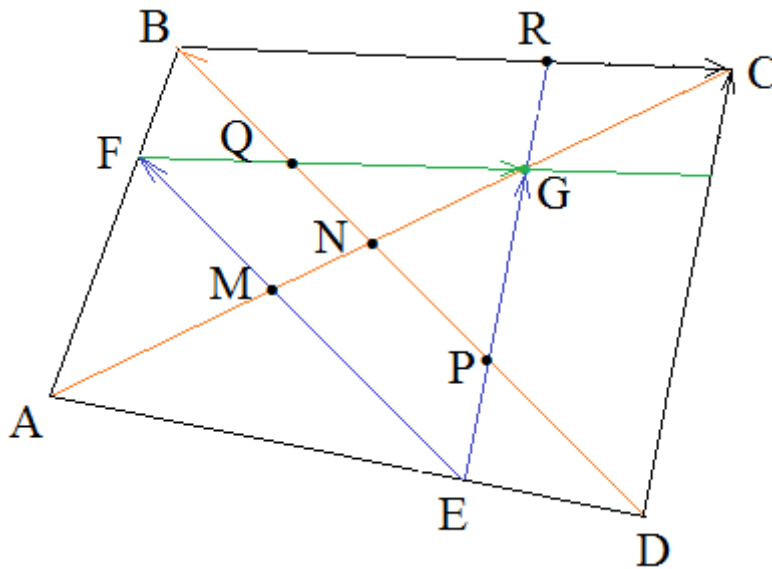


Answer on Question #70211 – Math – Geometry

Question

ABCD is a plane quadrilateral and E is any point on AD. EF is drawn parallel to DB to meet AB in F, and EG is drawn parallel to DC to meet AC in G. Prove that FG is parallel to BC.

Solution



$$\overrightarrow{EF} \parallel \overrightarrow{BD} \Rightarrow \triangle AFE \sim \triangle ABD$$

Let

$$\frac{FE}{BD} = \frac{AF}{AB} = \frac{AE}{AD} = k$$

$$\overrightarrow{ER} \parallel \overrightarrow{CD} \Rightarrow \triangle AGE \sim \triangle ACD$$

Let

$$\frac{EG}{DC} = \frac{AG}{AC} = \frac{AE}{AD} = m$$

Then  $m = k$

We have the vectors  $\overrightarrow{EF}$ ,  $\overrightarrow{DB}$ ,  $\overrightarrow{EG}$ ,  $\overrightarrow{DC}$ ,  $\overrightarrow{FG}$ , and  $\overrightarrow{BC}$ .

Then

$$\overrightarrow{FG} = \overrightarrow{EG} - \overrightarrow{EF}$$

$$\overrightarrow{BC} = \overrightarrow{DC} - \overrightarrow{DB}$$

$$\overrightarrow{DB} = \frac{1}{k} \overrightarrow{EF}$$

$$\overrightarrow{DC} = \frac{1}{m} \overrightarrow{EG} = \frac{1}{k} \overrightarrow{EG}$$

$$\overrightarrow{BC} = \overrightarrow{DC} - \overrightarrow{DB} = \frac{1}{k} \overrightarrow{EG} - \frac{1}{k} \overrightarrow{EF} = \frac{1}{k} (\overrightarrow{EG} - \overrightarrow{EF}) = \frac{1}{k} \overrightarrow{FG}$$

The vectors  $\overrightarrow{BC}$  and  $\overrightarrow{FG}$  are collinear vectors. Hence,  $\overrightarrow{FG} \parallel \overrightarrow{BC}$ .