

Answer on Question #70186 – Math – Functional Analysis

Question

Show that a norm on a vector space X is a sublinear functional on X .

Solution

Let's $f(x) = \|x\|$. Obviously, $f(x)$ is a function from a vector space X to the scalar field \mathbb{R} .

1. $\forall x \in X, \forall a \in \mathbb{R}_+, f(ax) = \|ax\| = |a| \cdot \|x\| = a \cdot \|x\| = a \cdot f(x)$, due to the multiplicative property of a norm.
2. $\forall x, y \in X, f(x+y) = \|x+y\| \leq \|x\| + \|y\| = f(x) + f(y)$, because of the triangle inequality.

Correctness of the statement and both properties (positive homogeneity and subadditivity) were proved.