Answer on Question #70166 – Math – Calculus

Question

Find the fourier series of f(x) of the given interval: f(x)= 1/2 x belongs to (-1,0)
-1/2 x belongs to (0,1)
Is the function above even or odd?

Solution

$$f(x) = \begin{cases} \frac{1}{2}x, & -1 < x < 0 \\ -\frac{1}{2}x, & 0 < x < 1 \end{cases}$$

$T = 2, l = 1$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n cos\frac{\pi n x}{l} + b_n sin\frac{\pi n x}{l}\right)$$

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)dx = \frac{1}{l}\int_{-l}^{0} f(x)dx + \frac{1}{l}\int_{0}^{l} f(x)dx = \frac{1}{1}\int_{-1}^{0} \frac{1}{2}xdx + \frac{1}{1}\int_{0}^{1} (-\frac{1}{2}x)dx = -\frac{1}{4} - \frac{1}{4} = -\frac{1}{2}$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x) cos\frac{\pi n x}{l} dx = \int_{-1}^{0} \frac{1}{2}x cos\frac{\pi n x}{1} dx + \int_{0}^{1} (-\frac{1}{2})x cos\frac{\pi n x}{1} dx = \frac{2}{n^2\pi^2}$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x) sin\frac{\pi n x}{l} dx = \int_{-1}^{0} \frac{1}{2}x sin\frac{\pi n x}{1} dx + \int_{0}^{1} (-\frac{1}{2})x sin\frac{\pi n x}{1} dx = 0$$

$$f(x) = -\frac{1}{4} + \sum_{n=1}^{\infty} \left(\frac{2}{n^2\pi^2} cos(\pi n x)\right)$$

The function is even because $b_n = 0$.

Answer:

$f(x) = -\frac{1}{4} + \sum_{n=1}^{\infty} \left(\frac{2}{n^2\pi^2} cos(\pi n x)\right)$, the function is even.