Question

The height of a certain species 't' years after it was planted is given by

$$H = 20\ln(3t+2)+30$$

cm:

a) how tall was the shrub when it was planted.

b) how long will it take for the shrub to reach a height of 1 metre.

c) At what rate is the shrubs height changing.

i) 3 years after being planted.

ii) 10 years after being planted.

Solution

a) When the shrub was planted then $t=0$.

Then $H(0)=20\ln(3\cdot 0+2)+30=20\ln 2+30\approx 43.86$.

So, the shrub had a tall of 43.86 cm.

b) The shrub has a height of 1 metre for each t such that $H(t)=100$.

Then

$$20\ln(3t+2)+30=100,$$

$$20\ln(3t+2)=70,$$

$$\ln(3t+2)=\frac{7}{2},$$

$$3t+2=e^{\frac{7}{2}},$$

$$3t=e^{\frac{7}{2}}-2,$$

$$t=\frac{e^{\frac{7}{2}}-2}{3}\approx 10.372.$$

So,

$t\approx 10.372$ years after planting, the bush will have a height of 1 meter.

c) The rate of the shrubs height changing is

$$\frac{dH}{dt} = \frac{20}{3t+2} \cdot 3 + 0 = \frac{60}{3t+2}.$$

i) $\left.\dfrac{dH}{dt}\right|_{t=3} = \dfrac{60}{3 \cdot 3 + 2} = \dfrac{60}{11}$.

ii) $\left.\dfrac{dH}{dt}\right|_{t=10} = \dfrac{60}{3 \cdot 10 + 2} = \dfrac{60}{32} = \dfrac{15}{8}$.