Answer on Question #70141 – Math – Calculus

Question

How do I prove this statement using properties of numbers: If a < 0, b < 0, then √ab ≤ −(a+b)/2?

Solution

Given

$$\sqrt{ab} \leq \frac{-a-b}{2}$$

Notice the following property. If one proves $c^2 \leq d^2$, where

$$c = \sqrt{ab} > 0, \quad d = \frac{-a-b}{2} = -\frac{1}{2}(a+b) > 0, \ a < 0, b < 0,$$

then one will also show that $c \leq d$ because

$$c^2 - d^2 = (c-d)(c+d) \leq 0, c + d = \sqrt{ab} + \frac{-a-b}{2} > 0 \ (a < 0, b < 0).$$

Consider

$$\left(\sqrt{ab}\right)^2 \leq \left(\frac{-a-b}{2}\right)^2$$

$$ab \leq \frac{(a+b)^2}{4}$$

$$4ab \leq a^2 + 2ab + b^2$$

$$(a^2 - 2ab + b^2) \geq 0$$

$$(a-b)^2 \geq 0$$

which is true because the square of any number is greater or equal than zero, the equality is attained at $a = b$. Then

$$\sqrt{ab} \leq \frac{-a-b}{2}$$

is also true for $a < 0$, $b < 0$.