

Answer on Question #70132 – Math – Calculus

Question

Use intermediate value theorem to prove $x(x - 3)^2 = 3$ has a solution between 2 and 4.

Solution

Let us consider the function $f(x) = x(x - 3)^2 - 3, x \in [2, 4]$. Obviously f is continuous as the polynomial $x^3 - 6x^2 + 9x - 3$

(see <https://math.berkeley.edu/~kmill/math1afa2014/poly.pdf>).

Note that $f(2) = -1$, and $f(4) = 1$.

Then by using intermediate value theorem

(see https://en.wikipedia.org/wiki/Intermediate_value_theorem) we conclude that f takes all the intermediate values between -1 and 1 , i.e. there exist $s, 2 < s < 4: f(s) = 0$, i.e.

$\exists 2 < s < 4: s(s - 3)^2 - 3 = 0 \Leftrightarrow s(s - 3)^2 = 3$, i.e. the equation $x(x - 3)^2 = 3$ has a solution between 2 and 4. The statement is proved.