

Answer on Question #70127 – Math – Calculus

Question

For what value of k , is the function f , defined by,

$$f(x) = \begin{cases} \frac{\sin(1+x) - \sin(1-x)}{x}, & \text{when } x \neq 0 \\ k \cos(1), & \text{when } x = 0 \end{cases}$$

continuous at $x = 0$?

Solution

We have that

$$\begin{aligned} \sin(1+x) - \sin(1-x) &= 2 \sin\left(\frac{1+x - (1-x)}{2}\right) \cos\left(\frac{1+x + (1-x)}{2}\right) = \\ &= 2 \sin(x) \cos(1) \end{aligned}$$

Theorem

$$\lim_{x \rightarrow 0} \left(\frac{\sin(x)}{x} \right) = 1$$

Find the limit

$$\begin{aligned} \lim_{x \rightarrow 0} f(x) &= \lim_{x \rightarrow 0} \left(\frac{\sin(1+x) - \sin(1-x)}{x} \right) = \lim_{x \rightarrow 0} (2 \sin(x) \cos(1)) = \\ &= 2 \cdot 1 \cdot \cos(1) = 2 \cos(1) \end{aligned}$$

The function $f(x)$ is continuous at $x = 0$, if

$$\lim_{x \rightarrow 0} f(x) = f(0)$$

Therefore,

$$2 \cos(1) = k \cos(1) \Rightarrow k = 2.$$

Answer: $k = 2$.