Answer on Question #70126 – Math – Calculus

Question

find out for which value of k the function f, given by
f(x)={ (1-cos4x)/x^2 , when x≠0 ; k(2+sin^(2) x), when x=0
is continuous at x=0.

Solution

Definition: A function *f(x)* is continuous at *a* if $\lim_{x \to a} f(x) = f(a)$.

Thus, we first find $\lim_{x \to 0} f(x)$

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{1 - \cos 4x}{x^2} = \lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = 2\lim_{x \to 0} \left(\frac{\sin 2x}{x}\right)^2$$

$$= 8\lim_{x \to 0} \left(\frac{\sin 2x}{2x}\right)^2 = 8\left(\lim_{x \to 0} \left(\frac{\sin 2x}{2x}\right)\right)^2 = 8 \cdot 1 = 8$$

Then we find f(0)

$f(0) = k(2 + \sin^2 0) = 2k$

The function *f(x)* is continuous at x=0 if $\lim_{x \to 0} f(x) = f(0)$, that is $8 = 2k$ => $k = 4$

Answer: The function f(x) is continuous at x=0 if k=4.