Answer on Question #70088 – Math – Differential Equations

Question

Show that the function

$\boldsymbol{i})\ u(x,t)\ = A(x+ct)^3$ is a solution of the one-dimensional wave equation.

$\boldsymbol{ii})\ u(x,t)\ =\ (e)^{-\mu\, t}\ \sin x$ is a solution of the one-dimensional heat equation.

Solution:

i)

$$u = (x_1 t) = A(x+ct)^3$$

The one-dimensional wave equation

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial u}{\partial t} = 3cA(x+ct)^2$$

$$\frac{\partial^2 u}{\partial t^2} = 6c^2 A(x+ct)$$

$$\frac{\partial u}{\partial x} = 3A(x+ct)^2$$

$$\frac{\partial^2 u}{\partial x^2} = 6A(x+ct)$$

So

$$6c^2 A(x+ct) = 6c^2 A(x+ct)$$

ii)

$$u(x,t) =\ (e)^{-\mu\, t}\ \sin x$$

The one-dimensional heat equation

$$\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2}$$

$$\frac{\partial u}{\partial t} = -\mu e^{-\mu t}\ \sin x$$

$$\frac{\partial u}{\partial x} = e^{-\mu t}\ \cos x$$

$$\frac{\partial^2 u}{\partial x^2} = -e^{-\mu t}\ \sin x$$

So

$$-\mu e^{-\mu t}\ \sin x = -\mu e^{-\mu t}\ \sin x$$