Answer on Question #70070 Math / Differential Equations

Solve the initial value problem: $\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 9x = 0,\ x(0) = 6,\ x'(0) = -1.$

Solution:

Let us consider the initial value problem

$$\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 9x = 0, \quad x(0) = 6, \quad x'(0) = -1$$

We will seek for its solution in the form

$$x(t) = e^{mt}.$$

Thus

$$\frac{\mathrm{dx}}{\mathrm{dt}} = \mathrm{me}^{\mathrm{mt}}, \qquad \frac{\mathrm{d}^2\mathrm{x}}{\mathrm{dt}^2} = \mathrm{m}^2\mathrm{e}^{\mathrm{mt}}.$$

Next, we will substitute these expressions into differential equation. We obtain algebraic equation

$$\mathrm{m}^2\mathrm{e}^{\mathrm{mt}} - 6\mathrm{me}^{\mathrm{mt}} + 9\mathrm{e}^{\mathrm{mt}} = 0,$$

$$\mathrm{m}^2 - 6\mathrm{m} + 9 = 0,$$

$$(m-3)^2 = 0,$$

$$m_1 = m_2 = 3.$$

So, the solution of the differential equation

$$x(t) = Ae^{3t} + Bte^{3t}.$$

The initial conditions give

$$x(0) = A = 6,$$

$$x'^{(0)} = 3A + B = -1, \qquad \rightarrow \ B = -19.$$

Finally

$$x(t) = 6e^{3t} - 19te^{3t}.$$

Answer: $x(t) = 6e^{3t} - 19te^{3t}$.