Answer on Question #70069-Math-Differential Equations

Solve the following ordinary differential equations:

i) dy/dx + 4xy = x,

Solution

$$\frac{dy}{dx} = x - 4xy = x(1 - 4y)$$

$$\frac{dy}{1 - 4y} = x\,dx$$

Integrating both sides,

$$\left(\frac{1}{4}\right) ln(1 - 4y) = -\left(\frac{x^2}{2}\right) + c$$

$$ln(1 - 4y) = -2x^2 + 4c$$

$$C = 4c$$

$$y = \frac{1 + e^{-2x^2 + C}}{4}$$

(2) d^2y/dx^2 + 4 dy/dx - 12 y =cos 2x

Solution

1) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} - 12\,y = 0$

$$y = e^{\lambda x}$$

$$\lambda^2 + 4\lambda - 12 = 0$$

$$\lambda_1 = -6, \lambda_2 = 2.$$

$$y = c_1 e^{-6x} + c_2 e^{2x}.$$

2)

$$y^* = A\cos 2x + B\sin 2x$$

$$y^{*\prime} = -2A\sin 2x + 2B\cos 2x$$

$$y^{*\prime\prime} = -4A\cos 2x - 4B\sin 2x$$

So,

$$-4A\cos 2x - 4B\sin 2x + 4(-2A\sin 2x + 2B\cos 2x) - 12(A\cos 2x + B\sin 2x) = \cos 2x$$

We have

$$-4A + 8B - 12A = 1$$

$$-4B - 8A - 12B = 0$$

$$8A = -16B$$

$$A = -2B.$$

$$-4(-2B) + 8B - 12(-2B) = 1$$

$$8B + 8B + 24B = 1$$

$$B = \frac{1}{40}$$

$$A = -2\frac{1}{40} = -\frac{1}{20}$$

The solution is

$$y = c_1 e^{-6x} + c_2 e^{2x} - \frac{1}{20}\cos 2x + \frac{1}{40}\sin 2x.$$