Answer on Question #70008 – Math – Statistics and Probability

Question

A random sample of 700 units from a large consignment showed that 200 were damaged. Find 95% confidence interval for the proportion of damaged unit in the consignment.

Solution

First, let's find the sample proportion:

$$\hat{p} = \frac{m}{n} = \frac{200}{700} = \frac{2}{7} \approx 0.285714.$$

Second, let's find the standard error of the sample proportion:

$$\sigma = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{\frac{2}{7}\left(1-\frac{2}{7}\right)}{700}} = \sqrt{\frac{\frac{10}{49}}{700}} = \sqrt{\frac{1}{3430}} \approx 0.017075.$$

Error in the sample proportion is believed to follow a normal distribution with σ computed above. For a two-tailed confidence interval with the level of confidence of 95%, we need corresponding z-score from a table:

$$z_{\alpha/2} = z_{(1-0.95)/2} = z_{0.025} = 1.96.$$

Then the margin of error is

$$z_{0.025}\sigma = 1.96 \times 0.017075 = 0.033467.$$

Finally, the 95% confidence interval is

$$(\hat{p} - z_{0.025}\sigma, \hat{p} + z_{0.025}\sigma) = (0.285714 - 0.033467, 0.285714 + 0.033467)$$
$$= (0.252247, 0.319181).$$

Answer: $(0.252247, 0.319181)$.