

Answer on Question #70003 – Math – Linear Algebra

Question

Obtain a unit vector perpendicular to the plane of the vectors

$\vec{a} = 3\hat{i} - 4\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j} + 3\hat{k}$.

Solution

$$\vec{a} = 3\vec{i} - 4\vec{j} + 2\vec{k}; \vec{b} = \vec{i} + \vec{j} + 3\vec{k};$$

$$\vec{a} (3; -4; 2);$$

$$\vec{b} (1; 1; 3).$$

Find the vector product (the cross product) of vectors \vec{a} and \vec{b} :

$$\vec{c} = [\vec{a} \times \vec{b}] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -4 & 2 \\ 1 & 1 & 3 \end{vmatrix} = \begin{vmatrix} -4 & 2 \\ 1 & 3 \end{vmatrix} \vec{i} - \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} \vec{j} + \begin{vmatrix} 3 & -4 \\ 1 & 1 \end{vmatrix} \vec{k} =$$

$$= -14\vec{i} - 7\vec{j} + 7\vec{k}, \quad \vec{c} (-14; -7; 7); \quad |\vec{c}| = \sqrt{(-14)^2 + (-7)^2 + 7^2} = 7\sqrt{6}.$$

It is known that $\vec{c} \perp \vec{a}, \vec{c} \perp \vec{b}$.

Find $\vec{d} \parallel \vec{c}; \quad |\vec{d}| = 1$:

$$\vec{d} = \frac{1}{7\sqrt{6}} \vec{c};$$

$\vec{d} \left(-\frac{\sqrt{6}}{3}; -\frac{\sqrt{6}}{6}; \frac{\sqrt{6}}{6} \right)$ is one of the required unit vectors.

Answer: $\vec{d} \left(-\frac{\sqrt{6}}{3}; -\frac{\sqrt{6}}{6}; \frac{\sqrt{6}}{6} \right)$.