

Answer on Question #69992 – Math – Algebra

Question

Write a polynomial function of minimum degree with real coefficients whose zeros include those listed. Write the polynomial in standard form.

8, -14, and $3 + 9i$

Solution

The other needed zero is $3 - 9i$ because complex zeros come in conjugate pairs.

Function in a form other than general (here being factored) will be

$$p(x) = (x - 8)(x + 14)(x - (3 + 9i))(x - (3 - 9i))$$

To keep "real coefficients" one needs a quadratic factor corresponding to the product of two complex factors with imaginary components.

$$\begin{aligned}(x - 3 - 9i)(x - 3 + 9i) &= x^2 - 3x + 9ix - 3x + 9 - 27i - 9ix + 27i - 81i^2 = \\ &= x^2 - 6x + 9 + 81 = x^2 - 6x + 90,\end{aligned}$$

hence

$$p(x) = (x - 8)(x + 14)(x^2 - 6x + 90).$$

Finally the standard form of the polynomial is

$$p(x) = x^4 - 58x^2 + 1212x - 10080$$

Answer: $p(x) = x^4 - 58x^2 + 1212x - 10080$.