Answer on Question #69778 – Math – Statistics and Probability

Question

A committee of four is to be formed from among 5 economists, 4 engineers, 2 statisticians and 1 doctor. Find the probabilities that
(i) Each profession is represented in the committee
(ii) Has at least one economists
(iii) Has a doctor as a member and three others
(iv) Has at least one engineer but no economists

Solution

$$5+4+2+1=12$$

The total number of selections is

$$\binom{12}{4}=\frac{12!}{(12-4)!\,4!}=\frac{12(11)(10)(9)}{1(2)(3)(4)}=495$$

(i) Each profession is represented in the committee
The following table shows the number of selections from each group:

Economists	Engineers	Statisticians	Doctors
1	1	1	1

Thus, a committee consisting of one profession from each field can be selected in

$$\binom{5}{1}\cdot\binom{4}{1}\cdot\binom{2}{1}\cdot\binom{1}{1}=5(4)(2)(1)=40$$

The probability that each profession is represented in the committee

$$P(A)=\frac{40}{495}=\frac{8}{99}$$

(ii) Has at least one economist
Find the probability that there is no economist in the committee

$$4+2+1=7$$

The suitable number of selections is

$$\binom{7}{4}=\frac{7!}{(7-4)!\,4!}=\frac{(7)(6)(5)}{1(2)(3)}=35$$

The probability that there is no economist in the committee

$$P(\overline{B})=\frac{35}{495}=\frac{7}{99}$$

The probability that there is at least one economist in the committee

$$P(B)=1-P(\overline{B})=1-\frac{7}{99}=\frac{92}{99}$$

(iii) Has a doctor as a member and three others

The following table shows the number of selections from each group:

Others	Doctors
1	1

$$5+4+2=11$$

Thus, a committee consisting of one profession from each field can be selected in

$$\binom{11}{3}\cdot\binom{1}{1}=\frac{11!}{(11-3)!\,3!}(1)=\frac{(11)(10)(9)}{1(2)(3)}=165$$

The probability that the committee has a doctor as a member and three others

$$P(C)=\frac{165}{495}=\frac{1}{3}$$

(iv) Has at least one engineer but no economists

The following table shows the number of selections from each group:

Economists	Engineers	Statisticians	Doctors
0	1	2	1
0	2	2	0
0	2	1	1
0	3	1	0
0	3	0	1
0	4	0	0

Thus, a committee having at least one engineer but no economists can be selected in

$$\binom{4}{1}\cdot\binom{2}{2}\cdot\binom{1}{1}+\binom{4}{2}\cdot\binom{2}{2}+\binom{4}{2}\cdot\binom{2}{1}\cdot\binom{1}{1}+\binom{4}{3}\cdot\binom{2}{1}+\binom{4}{3}\cdot\binom{1}{1}+\binom{4}{4}=$$

$$=4+\frac{4!}{(4-2)!\,(2)!}+\frac{4!}{(4-2)!\,(2)!}(2)+\frac{4!}{(4-3)!\,(3)!}(2)+\frac{4!}{(4-3)!\,(3)!}+1=$$

$$=4+6+12+8+4+1=35$$

The probability that the committee has at least one engineer but no economists

$$P(D)=\frac{35}{495}=\frac{7}{99}$$

Answer: i) $\frac{8}{99}$; **ii)** $\frac{92}{99}$; **iii)** $\frac{1}{3}$; **iv)** $\frac{7}{99}$.