Question

Find the area of an equilateral triangle inscribed in circle $x^2 + y^2 - 6x + 2y - 15 = 0$.

Solution

Let us present the equation of the given circle

$$x^2 + y^2 - 6x + 2y - 15 = 0$$

in standard form

(see http://www.mathwarehouse.com/geometry/circle/equation-of-a-circle.php):

$$(x^2 - 6x + 9) + (y^2 + 2y + 1) = 15 + 9 + 21 \Leftrightarrow (x - 3)^2 + (y + 1)^2 = 25.$$

Its radius r can be found from the following equation:

$$r^2 = 25 \Rightarrow r = 5.$$

Let A be an area of the inscribed triangle. Then we have:

$$A = \frac{3\sqrt{3}}{4} r^2 = \frac{3\sqrt{3}}{4} \cdot 25 = \frac{75\sqrt{3}}{4}.$$

(see https://en.wikipedia.org/wiki/Equilateral_triangle#Principal_properties).

Answer: $\frac{75\sqrt{3}}{4}$.