

Answer on Question #69436 – Math – Other

Question

A person standing at the crossing at two straight paths represented by the equations $2x-3y-4 = 0$ and $3x-4y-5 = 0$, wants to reach a path represented by $6x-7y+8 = 0$ in least time. Find the equations of path he should follow.

Solution

First, let's find where the person is standing. This is an intersection of two lines, which can be found by solving the system of equations representing these lines:

$$\begin{aligned} \begin{cases} 2x - 3y - 4 = 0 \\ 3x - 4y - 5 = 0 \end{cases} &\rightarrow \begin{cases} x = \frac{1}{2}(3y + 4) \\ 3x - 4y - 5 = 0 \end{cases} \rightarrow \begin{cases} x = \frac{1}{2}(3y + 4) \\ \frac{3}{2}(3y + 4) - 4y - 5 = 0 \end{cases} \rightarrow \begin{cases} x = \frac{1}{2}(3y + 4) \\ \frac{9}{2}y + 6 - 4y - 5 = 0 \end{cases} \\ &\rightarrow \begin{cases} x = \frac{1}{2}(3y + 4) \\ \frac{1}{2}y + 1 = 0 \end{cases} \rightarrow \begin{cases} x = \frac{1}{2}(3y + 4) \\ y = -2 \end{cases} \rightarrow \begin{cases} x = \frac{1}{2}(3(-2) + 4) \\ y = -2 \end{cases} \rightarrow \begin{cases} x = -1 \\ y = -2 \end{cases} \end{aligned}$$

The person is initially standing at $(x_0, y_0) = (-1, -2)$.

The shortest path to new 3rd path is perpendicular to it. For a line represented in form $ax + by + c = 0$ the perpendicular must follow the following form $-by + ax + d = 0$. 3rd path is represented by $6x - 7y + 8 = 0$, so perpendicular to it is $7x + 6y + d = 0$ for some unknown d . To find d let's substitute initial place into equation – this path must contain it:

$$7x_0 + 6y_0 + d = 0 \rightarrow 7(-1) + 6(-2) + d = 0 \rightarrow -19 + d = 0 \rightarrow d = 19.$$

The person must follow the path represented by $7x + 6y + 19 = 0$.

Answer: $7x + 6y + 19 = 0$.