Answer on Question #69421 – Math – Topology

Question

true/false? prove.

1/2 is a limit of the interval]− 2.5,1.5[

Solution

Recall that a limit point of a set S in a topological space X is a point x that can be "approximated" by points of S in the sense that every neighbourhood of x with respect to the topology on X also contains a point of S other than x itself.

Let U be arbitrary neighbourhood of $\frac{1}{2}$. There is $\varepsilon > 0$ such that $\left(\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon\right) \subseteq U$. Put $a = \max\left\{-2.5, \frac{1}{2} - \varepsilon\right\}$. Note that, $a < \frac{1}{2}$. Consider a point x such that $a < x < \frac{1}{2}$ (by example $x = \dfrac{a + \frac{1}{2}}{2}$). Since $a < x < \frac{1}{2}$, then $x \in \left(\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon\right) \subseteq U$ and $-2.5 < x < \frac{1}{2} < 1.5$, i.e. $x \in (-2.5,1.5)$.

This means that $\frac{1}{2}$ is a limit of the interval $(-2.5,1.5)$.

Answer: True.