

Question #6934 One box contains 3 red balls, white balls, and one blue ball. A second box contains one red ball, 2 white balls, and 3 blue balls. a. If one ball is selected at random from each box. Find the probability that the balls will be of the same color. b. If the balls in the two boxes are mixed together in a single box and then a sample of 3 is drawn. Find the probability that all 3 colors are represented both when sampling with replacement and when sampling without replacement.

Solution. The number of white balls in the first box is not said, so I assume that is equal to 3.

a) $P(2 \text{ selected balls are of the same color}) = P(\text{the ball from the first is red and the ball from the second is red}) + P(\text{the ball from the first is white and the ball from the second is white}) + P(\text{the ball from the first is blue and the ball from the second is blue})$. Selecting balls from different boxes is independent, hence the last sum equals to $3/7 \cdot 1/6 + 3/7 \cdot 2/6 + 1/7 \cdot 3/6 = 12/42 = 2/7$.

Answer. $2/7$. b) **without replacement** Note, that there are 4 red balls, 5 white balls and 4 blue balls after the mixing. Using classical definition of probability, one can get that $P(\text{all 3 colors are represented}) = 3! \frac{4 \cdot 5 \cdot 4}{13^3}$. Multiplier $3!$ indicates that look after the order of balls of different colors.

Answer. $3! \frac{4 \cdot 5 \cdot 4}{13^3}$.

a) **with replacement** $3! \cdot \frac{4 \cdot 5 \cdot 4}{13 \cdot 12 \cdot 11}$. Here, we have 12 balls after the first sampling, 11 balls after the second sampling. The probability to take balls in the order (red, white, blue) is $\frac{4 \cdot 5 \cdot 4}{13 \cdot 12 \cdot 11}$, multiplier $3!$ reflects all possible permutations of the colors.

Answer. $3! \cdot \frac{4 \cdot 5 \cdot 4}{13 \cdot 12 \cdot 11}$.