Answer on Question #69338 – Math – Algebra

Question

$$3x^2 + 1 > |x - 3| \qquad (1)$$

Solution

Because

$$|x-3| = \begin{cases} x-3 \text{ when } x \geq 3, \\ -x+3 \text{ when } x < 3, \end{cases}$$

we consider the following cases:

a) $x - 3 \geq 0, x \geq 3$

$3x^2 + 1 > x - 3$

$3x^2 - x + 4 > 0$

$D = (-1)^2 - 4 \cdot 3 \cdot 4 = 1 - 48 = -47 < 0, \quad a = 3 > 0$, hence

$x \in (-\infty, +\infty)$.

It follows from $x \geq 3$ and $x \in (-\infty, +\infty)$ that $x \in [3, +\infty)$.

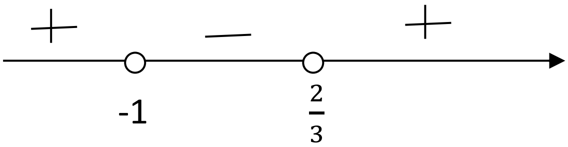
b) $x - 3 < 0, \quad x < 3$

$3x^2 + 1 > 3 - x,$

$3x^2 + x - 2 > 0,$

$D = 1^2 - 4 \cdot 3 \cdot (-2) = 25,$

$$x_{1,2} = \frac{-1 \pm 5}{2 \cdot 3} = \frac{-1-5}{6}; \frac{-1+5}{6} = -1; \frac{2}{3}$$



hence $x \in (-\infty, -1) \cup \left(\frac{2}{3}, +\infty\right)$.

It follows from $x < 3$ and $x \in (-\infty, -1) \cup \left(\frac{2}{3}, +\infty\right)$ that $x \in (-\infty, -1) \cup \left(\frac{2}{3}, 3\right)$.

Taking solutions in cases a) and b) one gets the general solution of the inequality (1):

$$\left[\begin{array}{c} x \in (-\infty, -1) \cup \left(\frac{2}{3}, 3\right) \\ x \in [3, +\infty) \end{array}\right. \rightarrow x \in (-\infty, -1) \cup \left(\frac{2}{3}, +\infty\right)$$

Answer: $x \in (-\infty, -1) \cup \left(\frac{2}{3}, +\infty\right)$.