**Answer on Question #69232 – Math – Statistics and Probability**

**Question**

Given that $P(X=2)=9P(X=4)+90P(X=6)$ for a Poisson distribution variate $X$. Find
$\boldsymbol{i})\ \ P(X<2)$

$\boldsymbol{ii})\ \ P(X>4)$

**iii**) $P(X \geq 1)$

**Solution**

The probability of observing x events in a given interval is given by

$$P(X=x)=e^{-\lambda}\frac{\lambda^x}{x!}\ , x=0,1,2,3,\ldots;\ \lambda>0$$

Given $P(X=2)=9P(X=4)+90P(X=6)$

$$e^{-\lambda}\frac{\lambda^2}{2!}=9e^{-\lambda}\frac{\lambda^4}{4!}+90e^{-\lambda}\frac{\lambda^6}{6!}$$

$$\frac{90}{720}\lambda^4+\frac{9}{24}\lambda^2-\frac{1}{2}=0$$

$$\lambda^4+3\lambda^2-4=0$$

$$(\lambda^2-1)(\lambda^2+4)=0$$

$$\lambda=1\ or\ \lambda=-1\ \ or\ \lambda^2=-4;\ \lambda>0$$

Hence $\lambda=1$.

$\boldsymbol{i})\ \ P(X<2)=P(X=0)+P(X=1)=e^{-\lambda}\frac{\lambda^0}{0!}+e^{-\lambda}\frac{\lambda^1}{1!}=e^{-1}+e^{-1}=$
$=2e^{-1}\approx 0.735759$

$\boldsymbol{ii})\ \ P(X>4)=$
$=1-\big(P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)\big)=$
$=1-\left(e^{-\lambda}\frac{\lambda^0}{0!}+e^{-\lambda}\frac{\lambda^1}{1!}+e^{-\lambda}\frac{\lambda^2}{2!}+e^{-\lambda}\frac{\lambda^3}{3!}+e^{-\lambda}\frac{\lambda^4}{4!}\right)=$
$=1-\left(e^{-1}+e^{-1}+e^{-1}\frac{1}{2}+e^{-1}\frac{1}{6}+e^{-1}\frac{1}{24}\right)=1-\frac{65}{24}e^{-1}\approx 0.003660$

**iii**) $P(X\geq 1)=1-P(X=0)=1-e^{-\lambda}\frac{\lambda^0}{0!}=1-e^{-1}\approx 0.632121$

**Answer: i)** $2e^{-1}$; **ii)** $1-\frac{65}{24}e^{-1}$; **iii)** $1-e^{-1}$.