

## Answer on Question #69162 – Math – Algebra

### Question

1. Find the values of  $a$  and  $b$  in each of the following:

$$\text{i) } \frac{5 + 2\sqrt{3}}{7 + 4\sqrt{3}} = a - 6\sqrt{3}$$

### Solution

$$\frac{5 + 2\sqrt{3}}{7 + 4\sqrt{3}} = \frac{5 + 2\sqrt{3}}{7 + 4\sqrt{3}} \cdot \frac{7 - 4\sqrt{3}}{7 - 4\sqrt{3}} = \frac{35 + 14\sqrt{3} - 20\sqrt{3} - 24}{(7)^2 - (4\sqrt{3})^2} = \frac{11 - 6\sqrt{3}}{49 - 48} =$$

$$= 11 - 6\sqrt{3} = a - 6\sqrt{3};$$

$$a = 11.$$

**Answer:**  $a = 11$ .

### Question

$$\text{ii) } \frac{3 - \sqrt{5}}{3 + 2\sqrt{5}} = a\sqrt{5} - \frac{9}{11}$$

### Solution

$$\frac{3 - \sqrt{5}}{3 + 2\sqrt{5}} = \frac{3 - \sqrt{5}}{3 + 2\sqrt{5}} \cdot \frac{3 - 2\sqrt{5}}{3 - 2\sqrt{5}} = \frac{9 - 3\sqrt{5} - 6\sqrt{5} + 10}{(3)^2 - (2\sqrt{5})^2} = \frac{19 - 9\sqrt{5}}{9 - 20} =$$

$$= \frac{9}{11}\sqrt{5} - \frac{19}{11} = a\sqrt{5} - \frac{9}{11};$$

$$a\sqrt{5} = \frac{9}{11}\sqrt{5} - \frac{19}{11} + \frac{9}{11};$$

$$a\sqrt{5} = \frac{9}{11}\sqrt{5} - \frac{10}{11};$$

$$a = \frac{9}{11} - \frac{10}{11\sqrt{5}};$$

$$a = \frac{9}{11} - \frac{10}{55}\sqrt{5};$$

$$a = \frac{9}{11} - \frac{2}{11}\sqrt{5}.$$

**Answer:**  $a = \frac{9}{11} - \frac{2}{11}\sqrt{5}$

### Question

$$\text{iii) } \frac{\sqrt{2} + \sqrt{3}}{3\sqrt{2} - 2\sqrt{3}} = 2 - b\sqrt{6}$$

### Solution

$$\frac{\sqrt{2} + \sqrt{3}}{3\sqrt{2} - 2\sqrt{3}} = \frac{\sqrt{2} + \sqrt{3}}{3\sqrt{2} - 2\sqrt{3}} \cdot \frac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} + 2\sqrt{3}} = \frac{3(2) + 3\sqrt{3(2)} + 2\sqrt{2(3)} + 2(3)}{(3\sqrt{2})^2 - (2\sqrt{3})^2} =$$

$$= \frac{12+5\sqrt{6}}{18-12} = \frac{12+5\sqrt{6}}{6} = 2 + \frac{5}{6}\sqrt{6} = 2 - b\sqrt{6};$$

$$b = -\frac{5}{6}.$$

**Answer:**  $b = -\frac{5}{6}$ .

### Question

iv)  $\frac{7 + \sqrt{5}}{7 - \sqrt{5}} - \frac{7 - \sqrt{5}}{7 + \sqrt{5}} = a + \frac{7}{11}\sqrt{5}b$

### Solution

$$\begin{aligned} \frac{7 + \sqrt{5}}{7 - \sqrt{5}} - \frac{7 - \sqrt{5}}{7 + \sqrt{5}} &= \frac{(7 + \sqrt{5})^2 - (7 - \sqrt{5})^2}{(7 + \sqrt{5})(7 - \sqrt{5})} = \\ &= \frac{49 + 14\sqrt{5} + 5 - 49 + 14\sqrt{5} - 5}{(7)^2 - (\sqrt{5})^2} = \frac{28\sqrt{5}}{49 - 5} = \frac{7}{11}\sqrt{5} = a + \frac{7}{11}\sqrt{5}b; \end{aligned}$$

$$a = 0, b = 1.$$

**Answer:**  $a = 0, b = 1$ .

### Question

2. If  $a = 2 + \sqrt{3}$ , then find the value of  $a - 1/a$ .

### Solution

$$\begin{aligned} a - \frac{1}{a} &= 2 + \sqrt{3} - \frac{1}{2 + \sqrt{3}} = 2 + \sqrt{3} - \frac{1}{2 + \sqrt{3}} \cdot \frac{2 - \sqrt{3}}{2 - \sqrt{3}} = \\ &= 2 + \sqrt{3} - \frac{2 - \sqrt{3}}{(2)^2 - (\sqrt{3})^2} = 2 + \sqrt{3} - (2 - \sqrt{3}) = 2 + \sqrt{3} - 2 + \sqrt{3} = 2\sqrt{3} \end{aligned}$$

**Answer:**  $2\sqrt{3}$ .

### Question

3. If  $a = 3 + \sqrt{5}/2$ , then find the value of  $a^2 - 1/a^2$ .

### Solution

$$a^2 = \left(3 + \frac{\sqrt{5}}{2}\right)^2 = 9 + 3\sqrt{5} + \frac{5}{4} = \frac{41 + 12\sqrt{5}}{4}$$

$$\frac{1}{a^2} = \frac{1}{\frac{41 + 12\sqrt{5}}{4}} = \frac{4}{41 + 12\sqrt{5}} \cdot \frac{41 - 12\sqrt{5}}{41 - 12\sqrt{5}} = 4 \cdot \frac{41 - 12\sqrt{5}}{(41)^2 - (12\sqrt{5})^2} =$$

$$= 4 \cdot \frac{41 - 12\sqrt{5}}{961} = \frac{164}{961} - \frac{48}{961}\sqrt{5}$$

$$a^2 - \frac{1}{a^2} = \frac{41}{4} + \frac{12\sqrt{5}}{4} - \frac{164}{961} + \frac{48}{961}\sqrt{5} =$$

$$= \frac{41(961) - 4(164)}{3844} + \frac{12(961) + 4(48)}{3844}\sqrt{5} = \frac{38745}{3844} + \frac{2931}{961}\sqrt{5}$$

**Answer:**  $\frac{38745}{3844} + \frac{2931}{961}\sqrt{5}$ .

### Question

4. Simplify:

$$\frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6} + \sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15} + 3\sqrt{2}}$$

### Solution

$$\frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6} + \sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15} + 3\sqrt{2}} =$$

$$= \frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}} \cdot \frac{\sqrt{10} - \sqrt{3}}{\sqrt{10} - \sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6} + \sqrt{5}} \cdot \frac{\sqrt{6} - \sqrt{5}}{\sqrt{6} - \sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15} + 3\sqrt{2}} \cdot \frac{\sqrt{15} - 3\sqrt{2}}{\sqrt{15} - 3\sqrt{2}} =$$

$$= \frac{7\sqrt{30} - 21}{10 - 3} - \frac{2\sqrt{30} - 10}{6 - 5} - \frac{3\sqrt{30} - 18}{15 - 18} =$$

$$= \sqrt{30} - 3 - 2\sqrt{30} + 10 + \sqrt{30} - 6 = 1$$

**Answer:** 1.