Answer on Question #69113 – Math – Real Analysis

Question

Check whether the function $f(x) = [x] + e^x$ is integrable in $[0, 3]$.

Solution

The function $f_1(x) = e^x$ is obviously integrable in $[0, 3]$ because e^x is continuous in $[0, 3]$ (see http://www.ams.jhu.edu/~prashant/continuous.pdf).

The function $f_2(x) = [x] = \begin{cases} 0, 0 \le x < 1 \\ 1, 1 \le x < 2 \\ 2, 2 \le x < 3 \\ 3, x = 3 \end{cases}$ is obviously monotonic in $[0, 3]$. But monotonic function is Riemann integrable

(see https://www.math.ucdavis.edu/~hunter/m125b/ch1.pdf p. 13), so $f_2(x)$ is integrable.

We see that original function $f(x)$ admits decomposition $f(x) = f_1(x) + f_2(x)$ where f_1 and f_2 are both integrable in $[0, 3]$. Since the sum of two integrable functions is integrable

(see https://www.math.ucdavis.edu/~hunter/m125b/ch1.pdf p. 16-17) we conclude that the function $f(x) = [x] + e^x$ is integrable in $[0, 3]$.

Answer: The function $f(x) = [x] + e^x$ is integrable in $[0, 3]$.