

Answer on Question #69112 – Math – Real Analysis

Question

Evaluate the limit as $n \rightarrow \infty$ of the sum
 $1/n [\sin(\pi/n) + \sin(2\pi/n) + \dots + \sin(2n\pi/n)]$

Solution

There is a mistake in the last term.

There should be $\frac{1}{n} \cdot \left(\sin\left(\frac{\pi}{n}\right) + \sin\left(\frac{2\pi}{n}\right) + \dots + \sin\left(\frac{n\pi}{n}\right) \right)$ instead of
 $\frac{1}{n} \cdot \left(\sin\left(\frac{\pi}{n}\right) + \sin\left(\frac{2\pi}{n}\right) + \dots + \sin\left(\frac{2n\pi}{n}\right) \right)$.

Sum $\frac{\pi}{n} \sum_{i=1}^n \sin\left(\frac{\pi i}{n}\right)$ is the right Riemann sum for the integral $\int_0^{\pi} \sin x dx$.

So $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \sin\left(\frac{\pi i}{n}\right) = \frac{1}{\pi} \int_0^{\pi} \sin x dx = -\frac{1}{\pi} \cos x \Big|_{x=0}^{x=\pi} = \frac{2}{\pi} \approx 0.6366$.

Answer: $\frac{2}{\pi} \approx 0.6366$.