Answer on Question #69102 - Math - Algebra

Question

1. If
$$\frac{\sqrt{5}-1}{\sqrt{5}+1} + \frac{\sqrt{5}+1}{\sqrt{5}-1} = a + b\sqrt{5}$$
, find a and b.

Solution

$$\begin{split} \frac{\sqrt{5}-1}{\sqrt{5}+1} + \frac{\sqrt{5}+1}{\sqrt{5}-1} &= \frac{\left(\sqrt{5}-1\right)\left(\sqrt{5}-1\right)}{\left(\sqrt{5}+1\right)\left(\sqrt{5}-1\right)} + \frac{\left(\sqrt{5}+1\right)\left(\sqrt{5}+1\right)}{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)} \\ &= \frac{\left(\sqrt{5}-1\right)\left(\sqrt{5}-1\right) + \left(\sqrt{5}+1\right)\left(\sqrt{5}+1\right)}{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)} \\ &= \frac{5-\sqrt{5}-\sqrt{5}+1+5+\sqrt{5}+\sqrt{5}+1}{\left(\sqrt{5}\right)^2-1^2} = \frac{12}{5-1} = \frac{12}{4} = 3 \end{split}$$

Answer: a = 3, b = 0.

Question

2. Simplify: $1/(1+\sqrt{2}-\sqrt{3})$ by rationalizing the denominator.

Solution

$$\frac{1}{1+\sqrt{2}-\sqrt{3}} = \frac{1(1-\sqrt{2}+\sqrt{3})}{(1+\sqrt{2}-\sqrt{3})(1-\sqrt{2}+\sqrt{3})}$$

$$= \frac{1-\sqrt{2}+\sqrt{3}}{1-\sqrt{2}+\sqrt{3}+\sqrt{2}-2+\sqrt{6}-\sqrt{3}+\sqrt{6}-3} = \frac{1-\sqrt{2}+\sqrt{3}}{2\sqrt{6}-4}$$

$$= \frac{1-\sqrt{2}+\sqrt{3}}{2(\sqrt{6}-2)} = \frac{(1-\sqrt{2}+\sqrt{3})(\sqrt{6}+2)}{2(\sqrt{6}-2)(\sqrt{6}+2)}$$

$$= \frac{\sqrt{6}+2-\sqrt{12}-2\sqrt{2}+\sqrt{18}+2\sqrt{3}}{2\left((\sqrt{6})^2-2^2\right)}$$

$$= \frac{\sqrt{6}+2-2\sqrt{3}-2\sqrt{2}+3\sqrt{2}+2\sqrt{3}}{2(2)} = \frac{2+\sqrt{6}+\sqrt{2}}{4}$$

Answer: $\frac{2+\sqrt{6}+\sqrt{2}}{4}$.

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