

## Answer on Question #68961 – Math – Linear Algebra

### Question

Check whether the following system of equations has a solution

$$x + y + 3z + w = 5$$

$$-x + y + z - 5w = 7$$

$$x + 2y + 5z - w = 5$$

### Solution

Augmented matrix

$$\begin{bmatrix} 1 & 1 & 3 & 1 & 5 \\ -1 & 1 & 1 & -5 & 7 \\ 1 & 2 & 5 & -1 & 5 \end{bmatrix}$$

Add row1 to row2 ( $R_2 \leftarrow R_2 + R_1$ )

$$\begin{bmatrix} 1 & 1 & 3 & 1 & 5 \\ 0 & 2 & 4 & -4 & 12 \\ 1 & 2 & 5 & -1 & 5 \end{bmatrix}$$

Subtract row1 from row3 ( $R_3 \leftarrow R_3 - R_1$ )

$$\begin{bmatrix} 1 & 1 & 3 & 1 & 5 \\ 0 & 2 & 4 & -4 & 12 \\ 0 & 1 & 2 & -2 & 0 \end{bmatrix}$$

Divide row2 by 2 ( $R_2 \leftarrow R_2/2$ )

$$\begin{bmatrix} 1 & 1 & 3 & 1 & 5 \\ 0 & 1 & 2 & -2 & 6 \\ 0 & 1 & 2 & -2 & 0 \end{bmatrix}$$

Subtract row2 from row1 ( $R_1 \leftarrow R_1 - R_2$ )

$$\begin{bmatrix} 1 & 0 & 1 & 3 & -1 \\ 0 & 1 & 2 & -2 & 6 \\ 0 & 1 & 2 & -2 & 0 \end{bmatrix}$$

Subtract row2 from row3 ( $R_3 \leftarrow R_3 - R_2$ )

$$\begin{bmatrix} 1 & 0 & 1 & 3 & -1 \\ 0 & 1 & 2 & -2 & 6 \\ 0 & 0 & 0 & 0 & -6 \end{bmatrix}$$

Add row3 to row2 ( $R_2 \leftarrow R_2 + R_3$ )

$$\begin{bmatrix} 1 & 0 & 1 & 3 & -1 \\ 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & -6 \end{bmatrix}$$

Divide row3 by  $-6$  ( $R_3 \leftarrow R_3/(-6)$ )

$$\begin{bmatrix} 1 & 0 & 1 & 3 & -1 \\ 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Add row3 to row1 ( $R_1 \leftarrow R_1 + R_3$ )

$$\begin{bmatrix} 1 & 0 & 1 & 3 & 0 \\ 0 & 1 & 2 & -2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{cases} x + z - w = 0 \\ y + 2z - 2w = 0 \\ 0 = 1 \end{cases}$$

The system is inconsistent and has no solution.

**Answer:** the system has no solution.