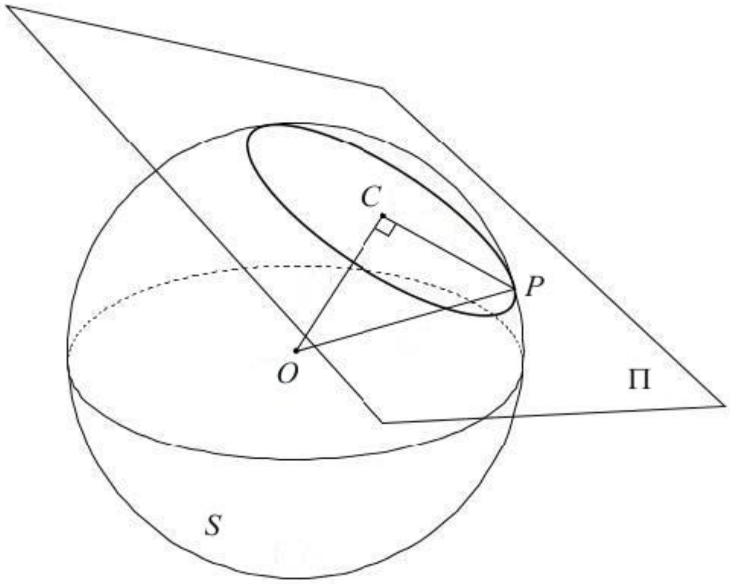
Answer on Question #68960 – Math – Linear Algebra

Question

Find the radius and the center of the circular section of the sphere $|r| = 17$ cut off by the plane $r \cdot (i + 2j + 2k) = 24$.

Solution



Let S be the sphere in $\mathbb{R}^3$ with center $O(0,0,0)$ and radius R, and let Π be the plane with equation $Ax + By + Cz = D$, so that $\vec{n} = (A, B, C)$ is a normal vector of Π.
If P is an arbitrary point on Π , the signed distance from the center of the sphere O to the plane Π is

$$\rho = \frac{\overrightarrow{P0} \cdot \vec{n}}{||\vec{n}\,||} = \frac{D}{\sqrt{A^2 + B^2 + C^2}}.$$

The intersection $S \cap \Pi$ is a circle if and only if $-R < \rho < R$, and in that case, the circle has radius $r_c = \sqrt{R^2 - \rho^2}$ and center

$$C = O + \rho \cdot \frac{\vec{n}}{||\vec{n}\,||} = \rho \cdot \frac{(A, B, C)}{\sqrt{A^2 + B^2 + C^2}}.$$

In our case:

$$|r| = 17;\ x + 2y + 2z = 24$$

$$S = \{(x, y, z): x^2 + y^2 + z^2 = 289\}, P = \{(x, y, z): x + 2y + 2z = 24\}.$$

$$\rho = \frac{24}{\sqrt{1^2 + 2^2 + 2^2}} = \frac{24}{3}.$$

$$r_c = \sqrt{R^2 - \rho^2} = \sqrt{17^2 - \left(\frac{24}{3}\right)^2} = 15.$$

$$C = \frac{24}{3} \cdot \frac{(1,2,2)}{\sqrt{1^2 + 2^2 + 2^2}} = \left(\frac{24}{9}, \frac{48}{9}, \frac{48}{9}\right).$$

Answer: 15; $\left(\frac{24}{9}, \frac{48}{9}, \frac{48}{9}\right)$.

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