Answer on Question #68959 – Math – Linear Algebra

Question

Find the equation of the line passing through the point $A(1,\ 0,-1)$ and parallel to the line joining $B(1,\ 2,\ 3)$ and $C(-1,\ 2,\ 0)$. Is it perpendicular to the line $(1\ +\ 3\alpha\ ,\ 0\ ,1\ -\ 2\alpha\ )$?

Solution

1) Note that the line passing through $B(1,\ 2,\ 3)$ and $C(-1,\ 2,\ 0)$ has the direction vector $\vec{s}=\overrightarrow{BC}=\overrightarrow{(x_C-x_B,\ y_C-y_B,\ z_C-z_B)}=\overrightarrow{(-1-1,\ 2-2,\ 0-3)}=\overrightarrow{(-2,\ 0,-3)};$

2) The equation of the line passing through the point $A(x_0,\ y_0,z_0)$ and parallel to the line with direction vector $\vec{s}=\overrightarrow{(m,\ n,\ p)}$ has the canonical form:

$$\frac{x-x_0}{m}=\frac{y-y_0}{n}=\frac{z-z_0}{p}$$

and the parametric form:

$$\begin{cases} x=x_0+m\alpha, \\ \mathrm{y}=\mathrm{y}_0+n\alpha, \\ z=z_0+p\alpha. \end{cases}$$

We have $A(1,\ 0,-1)$, $\vec{s}=\overrightarrow{(-2,\ 0,-3)}$, substituting one gets

$$\frac{x-1}{-2}=\frac{y-0}{0}=\frac{z-(-1)}{-3};$$

$$l:\ \frac{x-1}{-2}=\frac{y-0}{0}=\frac{z+1}{-3}.$$

3) The line $(1\ +\ 3\alpha,\ 0,1\ -\ 2\alpha\ )$ has the next parametric form:

$$\begin{cases} x=1\ +\ 3\alpha, \\ \mathrm{y}=0, \\ z=1\ -\ 2\alpha. \end{cases}$$

After reducing this form to the canonical equation

$$l_1:\frac{x-1}{3}=\frac{y-0}{0}=\frac{z-1}{-2}.$$

We note that the line has the direction vector $\vec{s}_1=\overrightarrow{(3,\ 0,-2)}$.

4) The line l with the direction vector s is perpendicular to the line l_1 with the direction vector s_1 iff the scalar product is

$$s\cdot s_1=0.$$

Note that

$$s\cdot s_1=m\cdot m_1+n\cdot n_1+p\cdot p_1=-2\cdot 3+0\cdot 0+(-3)\cdot(-2)=-6+6=0,$$

so line l is perpendicular to the line l_1.

Answer: $l:\ \frac{x-1}{-2}=\frac{y-0}{0}=\frac{z+1}{-3}$; it is perpendicular to the line $(1\ +\ 3\alpha,\ 0,\ 1\ -\ 2\alpha\ )$.