

Answer on Question #68869 – Math – Statistics and Probability

Question

1. A certain shop repairs both audio and video components. Let A denote the event that the next component brought in for repair is an audio component, and let B be the event that the next component is a compact disc player (so the event B is contained in A). Suppose that $P(A) = 0.6$ and $P(B) = 0.05$. What is $P(B|A)$?

Solution

We have that B is contained in A, then $A \cap B = B$ and

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(B)}{P(A)} = \frac{0.05}{0.6} = \frac{1}{12} \approx 0.0833$$

Answer: $P(B|A) = \frac{1}{12} \approx 0.0833$.

Question

2. Each of 12 refrigerators of a certain type has been returned to a distributor because of an audible, high-pitched, oscillating noise when the refrigerator is running. Suppose that 7 of these refrigerators have defective compressor and the other 5 have less serious problems. If the refrigerators are examined in random order, let X be the number among the first 6 examined that have a defective compressor. What is the probability that X exceeds its mean value by more than 1 standard deviation?

Solution

It is hypergeometric probability distribution with $N = 12$, $n = 6$, $M = 7$, $N - M = 5$. The distribution is given by

$$P(X = r) = \frac{\binom{M}{r} \binom{N - M}{n - r}}{\binom{N}{n}}$$

The expectation (mean) and variance of the Hypergeometric random variable are given by

$$E(X) = \mu = np \quad \text{and} \quad V(X) = np(1 - p) \frac{N - n}{N - 1},$$

$$\text{where } p = \frac{M}{N}$$

Hence

$$p = \frac{7}{12};$$

$$E(X) = \mu = 6 \left(\frac{7}{12} \right) = \frac{7}{2} = 3.5;$$

$$V(X) = \sigma^2 = 6 \binom{7}{12} \left(1 - \frac{7}{12}\right) \left(\frac{12-6}{12-1}\right) = \frac{35}{44}.$$

Standard deviation is

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{35}{44}} \approx 0.8919$$

The probability that X exceeds its mean value by more than 1 standard deviation is

$$P(X > \mu + \sigma) = P(X > 3.5 + 0.8919) = P(X > 4.3919) = P(X \geq 5) =$$

$$= P(X = 5) + P(X = 6) = \frac{\binom{7}{5} \binom{5}{6-5}}{\binom{12}{6}} + \frac{\binom{7}{6} \binom{5}{6-6}}{\binom{12}{6}} =$$

$$= \frac{\frac{7!}{5!(7-5)!}(5) + \frac{7!}{6!(7-6)!}(1)}{\frac{12!}{6!(12-6)!}} = \frac{\frac{7(6)}{1(2)}(5) + 7}{\frac{12(11)(10)(9)(8)(7)}{1(2)(3)(4)(5)(6)}} = \frac{4}{33} \approx 0.1212.$$

Answer: $P(X > \mu + \sigma) = \frac{4}{33} \approx 0.1212.$

Question

3. Evaluate $(x - 2)!(y - 4)$ if $x = 6$ and $y = 9$.

Solution

Substitute 6 for x and 9 for y

$$(6 - 2)!(9 - 4) = 4!(5) = 1(2)(3)(4)(5) = 120 = 5!$$

Answer: 120.